King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 254: Discrete Structures II
Second semester 2016-2017 (162)
Major Exam #1, Thursday March 9, 2017
Time: **120** Minutes

Name: ____

ID#: ____________________

Section: ___________________

Instructions:
1. The exam consists of 10 pages, including this page, containing 5 questions.
2. Answer all questions. **Show all the steps.**
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. **No calculators** or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	**100**	

Q1: **[20 points] Evaluate the following.**

(a) [10 points] $(59^8 \bmod 19)^4 \bmod 18$

(b) [10 points] $(1AE)_{16} \times (BBC)_{16} = (\qquad\qquad\qquad)_8$

Q2: [20 points] Solve the following questions

(a) [10 points] Prove that the product of any three consecutive integers is divisible by 6.

(b) [10 points] Show that a positive integer is divisble by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.

Q3: [20 points] Solve the following questions

(a) [10 points] Using the modular exponentiation algorithm, find 5^{37} mod 19.

(b) [10 points] Let x be a positive integer divisible by 5.
The *lcm* of x and $x + 5$ is 1530. Find x.

Q4: [20 points]

(a) [10 points] Solve the congruence: $185x \equiv 46 \pmod{253}$. Show all your steps.

(b) [10 points] Use the method of back substitution to solve the system of congruences:

$$x \equiv 5 \pmod{24},\ x \equiv 23 \pmod{30}.$$

Q5: [20 points]

(a) [10 points] Use Fermat's Little Theorem to find

(i) 7^{111} mod 23, and

(ii) 7^{111} mod 19.

(b) [10 points] Use the results of part (a) and the Chinese Remainder theorem to find $7^{111} \bmod 437$.